



8.1

Euler

$$\begin{cases} y' = x + y, 0 < x \leq 1 \\ y(0) = 1 \end{cases}$$

h=0.1

$$y = 2e^x - x - 1$$

8.2

Euler

$$\begin{cases} y' = x^2 + x - y \\ y(0) = 0 \end{cases}$$

h=0.1

y(0.5)

$$y = x^2 - x + 1 - e^{-x}$$





8.3

$$\begin{cases} y' = -y \\ y(0) = 1 \end{cases}$$

Euler

$$y_n = (1-h)^n, \quad y_n = \left(\frac{2-h}{2+h}\right)^n$$

$h \rightarrow 0$ $y(x) = e^{-x}$

8.4 $h=0.2$, R-K

$$(1) \begin{cases} y' = x + y, 0 < x \leq 1 \\ y(0) = 1 \end{cases}$$

$$(2) \begin{cases} y' = \frac{3y}{1+x}, 0 < x \leq 1 \\ y(0) = 1 \end{cases}$$





8.5

t R K

$$y_{n+1} = y_n + \frac{h}{2}(K_2 + K_3)$$

$$K_1 = f(x_n, y_n)$$

$$K_2 = f(x_n + th, y_n + thK_1)$$

$$K_3 = f(x_n + (1-t)h, y_n + (1-t)hK_1)$$

8.6

$$y' = \lambda y (\lambda < 0)$$

Euler

$$\left| 1 + \lambda h + \frac{(\lambda h)^2}{2} \right| \leq 1$$

R-K

$$\left| 1 + \lambda h + \frac{(\lambda h)^2}{2} + \frac{(\lambda h)^3}{6} + \frac{(\lambda h)^4}{24} \right| \leq 1$$



