



7



7

$$A \in R^{n \times n} \text{ (} C^{n \times n} \text{), } \lambda \in C, x,$$

$$Ax = \lambda x$$

$$\lambda, A, x, \lambda.$$

x

$$\varphi(\lambda) = \det(\lambda I - A) = \lambda^n + c_1 \lambda^{n-1} + \dots + c_{n-1} \lambda + c_n = 0$$

$$\varphi(\lambda) = 0 \quad n$$

$$\begin{cases} - (u_{xx} + u_{yy}) = \lambda u, (x, y) \in \Omega, \\ u = 0, (x, y) \in \partial\Omega \end{cases}$$



$$\Omega = \{ (x, y) : -1 < x, y < 1 \}, \partial\Omega \quad \Omega$$

$$\Delta x = \Delta y = h =$$

0.25,

$$\frac{1}{h^2} Bu = \lambda u,$$

B

u

5

$$\varphi \quad \lambda = 0$$



7.1

7.1 $A = (a_{ij}) \in R^{n \times n}, \lambda_i (i = 1, 2, \dots, n)$ A ,

(1) $\prod_{i=1}^n \lambda_i = \det(A)$

(2) $\sum_{i=1}^n \lambda_i = \sum_{i=1}^n a_{ii} = \text{tr}(A)$, A .

7.2 (Gershgorin) $A = (a_{ij}) \in C^{n \times n}, A$

$$\lambda \in \bigcup_{i=1}^n D_i,$$

D_i i :

$$D_i = \left\{ z : |z - a_{ii}| \leq \sum_{j=1, j \neq i}^n |a_{ij}| \right\}, i = 1, 2, \dots, n$$



λ 是 A 的特征值, $x \neq 0$

$$(\lambda I - A)x = 0$$

$$x = (x_1, x_2, \dots, x_n)^T, |x_i| = \max_k |x_k|, x_i \neq 0,$$

$$(\lambda - a_{ii})x_i = \sum_{j=1, j \neq i}^n a_{ij}x_j$$

$$|x_j / x_i| \leq 1 (j \neq i),$$

$$|\lambda - a_{ii}| \leq \sum_{j \neq i} |a_{ij}| |x_j / x_i| \leq \sum_{j \neq i} |a_{ij}|.$$

$$\lambda \in D_i.$$

λ 是 A 的特征值, $x \neq 0$. 7.2, A



7.1: A n , x ,

$$R(x) = \frac{(Ax, x)}{(x, x)}$$

x Rayleigh .

7.3 A n , ,

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n,$$

$$x_1, x_2, \dots, x_n,$$

(1) $x \in R^n, \lambda_n \leq R(x) \leq \lambda_1$

(2) $\lambda_1 = \max_{0 \neq x \in R^n} R(x) = R(x_1)$

(3) $\lambda_n = \min_{0 \neq x \in R^n} R(x) = R(x_n)$



$x \neq 0$

$$x = \sum_{i=1}^n \alpha_i x_i,$$

$$(x, x) = \sum_{i=1}^n \alpha_i^2 > 0,$$

$$\lambda_n \sum_{i=1}^n \alpha_i^2 \leq \sum_{i=1}^n \alpha_i^2 \lambda_i = (Ax, x) \leq \lambda_1 \sum_{i=1}^n \alpha_i^2$$

(1) (2) (3)

$$A \in C^{n \times n},$$

,

"A"

"

"A"

Hermite

$$A^H = A,$$

