



## 数值试验题2

2.1 编制分段线性插值和分段三次Hermite插值程序，对被插函数  $f(x)=1/(1+x^2)$ ，插值区间  $[a,b]$ ，分成10等分，求分段线性插值函数在各节点间中点处的值，并画出分段线性插值函数和  $y = f(x)$ 的图形。

2.2 给定数据如下表：

$x_i$	0	1	2	3	4	5	6	7	8	9	10
$y_i$	0.0	0.79	1.53	2.19	2.71	3.03	3.27	2.89	3.06	3.19	3.29

编制程序求三次样条插值函数在插值中点的样条函数值，并作点集  $\{x_i, y_i\}$  和样条插值函数的图形，满足的边界条件为

- (1)  $S'(0) = 0.8, S'(10) = 0.2$ .
- (2)  $S''(0) = S''(10) = 0$ .





**2.3** 对下列数据作三次多项式拟合，取权数  $w_i = 1$ ，给出拟合多项式的系数、平方误差并作离散数据  $\{x_i, y_i\}$  和拟合多项式的图形。

$x_i$	-1.0	-0.5	0.0	0.5	1.0	3.0	3.5
$y_i$	-4.447	-0.452	0.551	0.048	-0.447	0.549	4.552





下面6张是第1题





首先算出 $f(x)$ 的值如下:

```
>> x=[-5,-4,-3,-2,-1,0,1,2,3,4,5];y=1./(1+x.^2)
```

y =

```
    0.0385    0.0588    0.1000    0.2000    0.5000    1.0000    0.5000    0.2000  
    0.1000    0.0588    0.0385
```

分段线性插值

```
function l=f(x)
```

```
a=[-5,-4,-3,-2,-1,0,1,2,3,4,5];
```

```
b=[ 0.0385,0.0588,0.1000,0.2000,0.5000,1.0000,0.5000,0.2000,0.1000,0.0588,0  
.0385];
```

```
n=length(x);
```

```
for i=1:n-1
```

```
    l(i)=b(i)*(x(i)-a(i+1))/(a(i)-a(i+1))+b(i+1)*(x(i)-a(i))/(a(i+1)-a(i));
```

```
end
```





运行结果如下：

```
>> x=[-4.5,-3.5,-2.5,-1.5,-0.5,0.5,1.5,2.5,3.5,4.5];l=f(x)
```

```
l =
```

```
    0.0486    0.0794    0.1500    0.3500    0.7500    0.7500    0.3500    0.1500  
    0.0794    0.0486
```





## Hermite 插值

```
function l=f(x)
a=[-5,-4,-3,-2,-
1,0,1,2,3,4,5];m=[0.0148,0.0277,0.0600,0.1600,0.5000,0,
-0.5000,-0.1600,-0.0600,-0.0277,-0.0148]
b=[ 0.0385,0.0588,0.1000,0.2000,0.5000,1.0000,0.5000,0.2000,0.10
00,0.0588,0.0385];
n=length(x);
for i=1:n-1
    l(i)=(1+2*(x(i)-a(i+1))/(a(i)-a(i+1))*((x(i)-a(i))/(a(i+1)-a(i))).^2;
    n(i)=(x(i)-a(i+1))*((x(i)-a(i))/(a(i+1)-a(i))).^2;

    l(i)=b(i)*a(i)+m(i)*n(i)
end
```





$x=[-4.5,-3.5,-2.5,-1.5,-0.5,0.5,1.5,2.5,3.5,4.5];f(x)$

ans =

Columns 1 through 7

0.0486 0.0794 0.1500 0.3500 0.7500 0.7500 0.3500

Columns 8 through 10

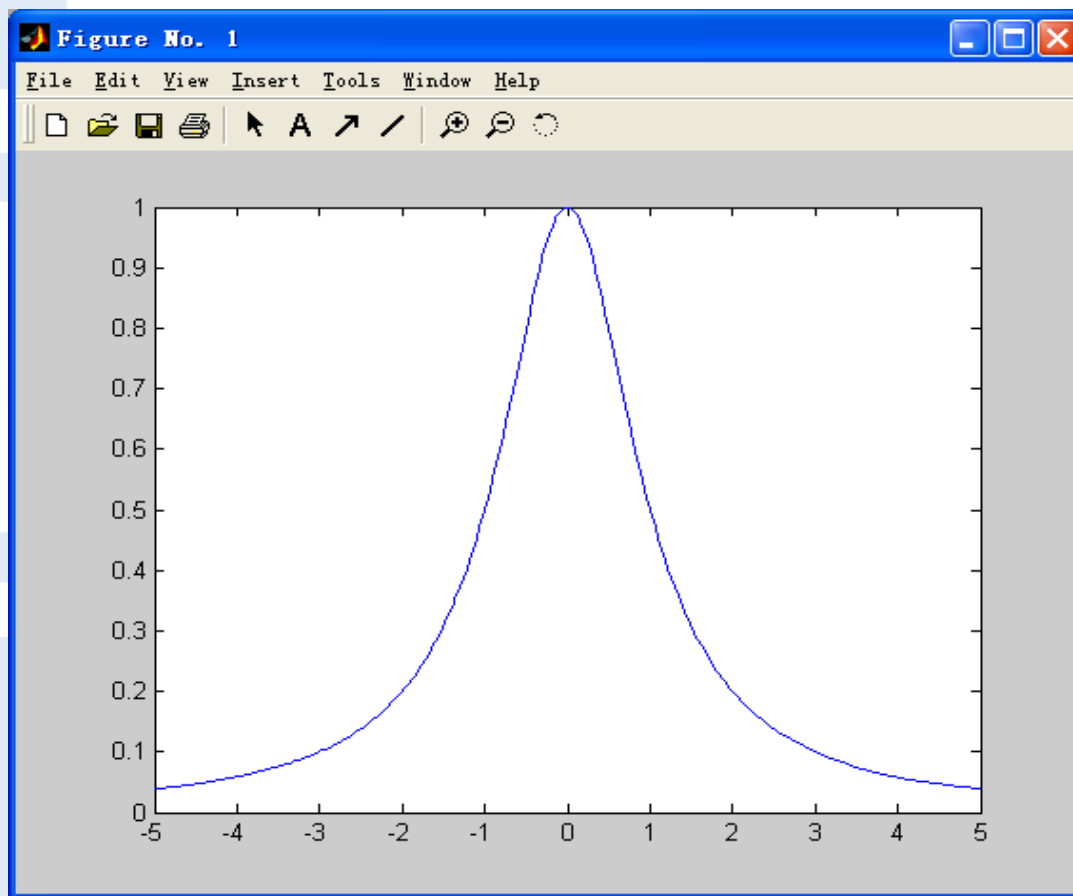
0.1500 0.0794 0.0486





输入: `>> fplot('1./(1+x.^2)',[-5,5])`

得 $f(x)$ 图形如下:



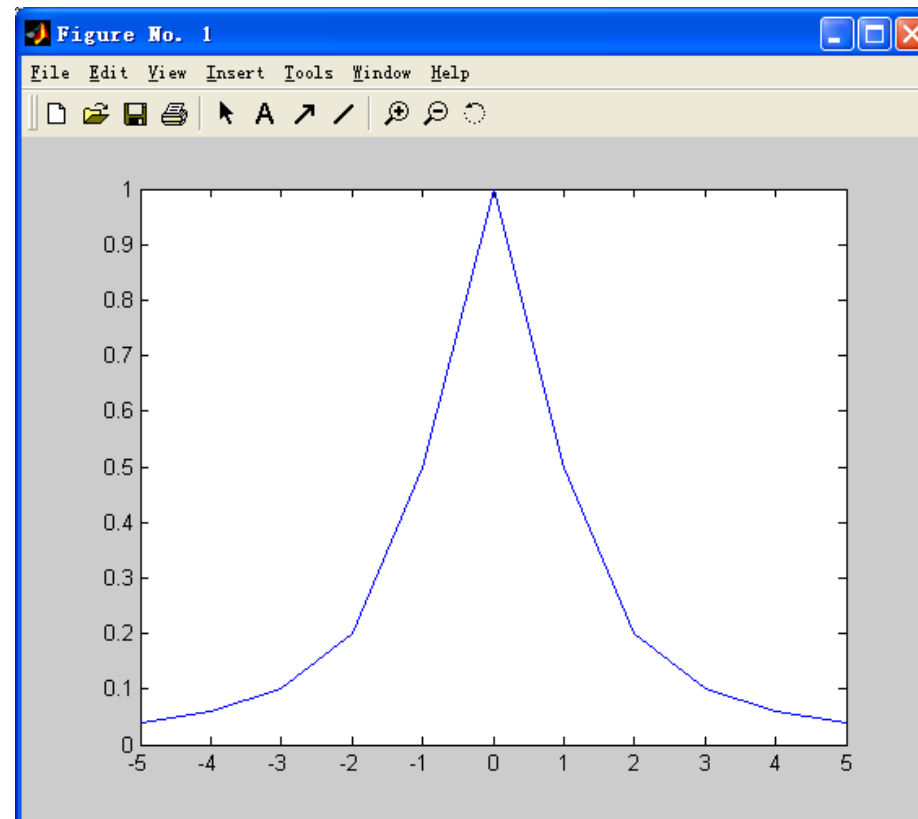




画图命令如下:

```
x=[-5,-4,-3,-2,-1,0,1,2,3,4,5];y=[ 0.0385,0.0588,0.1000,0.2000,0.5000,1.0000,0.5000,0.2000,0.1000,0.0588,0.0385];plot(x,y)  
>>
```

图形如下:





下面6张是第2题





```
function Sanciyangtiao( X, Y, a, b )
% a = f'(1), b = f'(n)
n = length( X );
for i = 1 : n-1
    h(i) = X(i+1)-X(i);
end
for j = 2 : n-1
    u(j) = h(j-1)/( h(j-1) + h(j) );
    t(j) = 1 - u(j);
    d(j) = 6*( (Y(j+1) - Y(j))/h(j) - (Y(j) - Y(j-1))/h(j-1) )/( h(j-1) + h(j) );
end
t(1) = 1; u(n) = 1;
d(1) = 6/h(1)*( (Y(2)-Y(1))/h(1) - a );
d(n) = 6/h(n-1)*( b - (Y(n)-Y(n-1))/h(n-1) );
for i = 1 : n
    A(i,i)=2; A(1,2)=1; A(n,n-1)=1;
    for j = 2 : n-1
        A(j,j-1)=u(j); A(j,j+1)=t(j);
        if abs(i-j)>1
            A(i,j) = 0;
        end
    end
end
end
```

程序(1)(a)



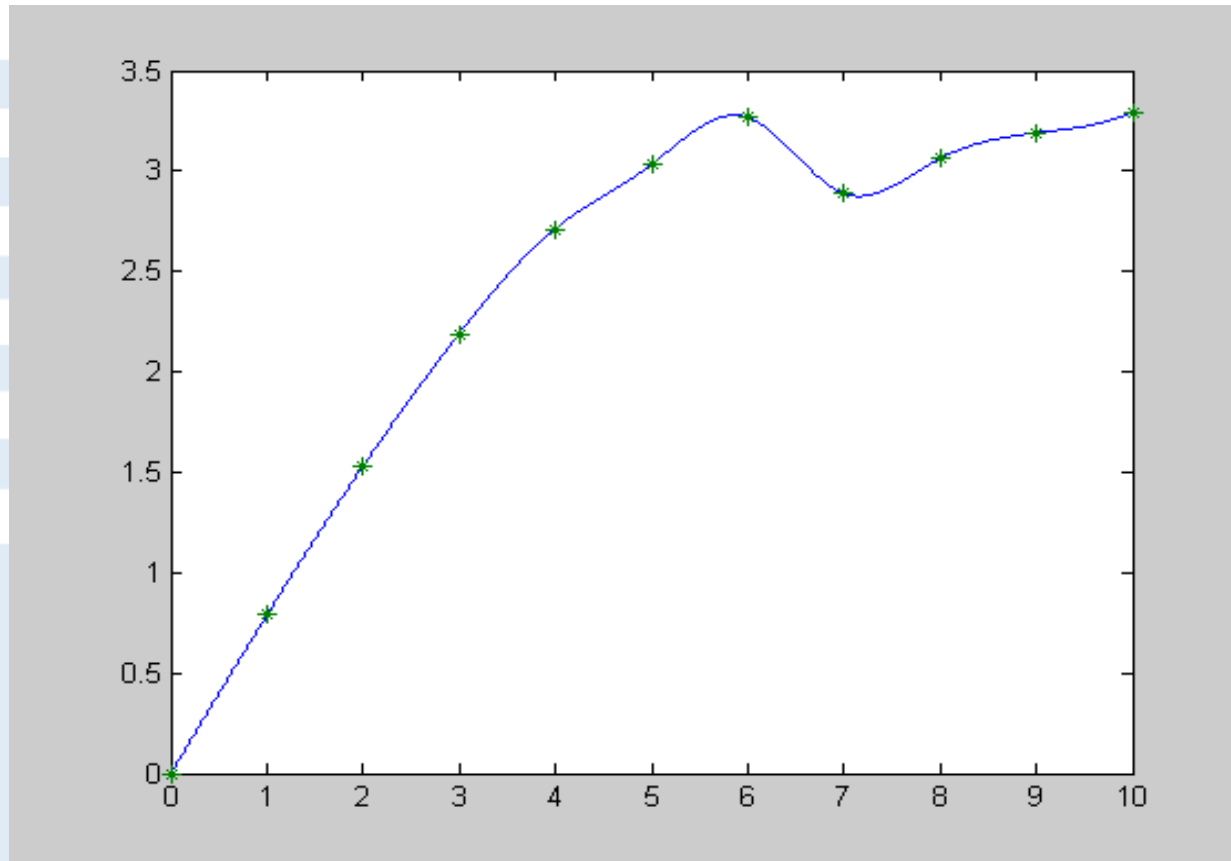


```
D = d';
N = A\D;
M = N';
for i = 1 : n-1
    H(i)=(M(i+1)-M(i))/(6*h(i));
    l(i)=(M(i)*X(i+1)-M(i+1)*X(i))/(2*h(i));
    J(i)=(M(i+1)*X(i)^2-M(i)*X(i+1)^2+2*(Y(i+1)-Y(i))+h(i)^2/3*(M(i)-M(i+1)))/(2*h(i));
    K(i)=(M(i)*X(i+1)^3-M(i+1)*X(i)^3+(6*Y(i)-M(i)*h(i)^2)*X(i+1)-(6*Y(i+1)-
M(i+1)*h(i)^2)*X(i))/(6*h(i));
end
for i=1 : n-1
    Q(i)=M(i)*(X(i+1)-(X(i)+X(i+1))/2)^3/(6*h(i))+M(i+1)*((X(i)+X(i+1))/2-X(i))^3/(6*h(i))+Y(i) -
M(i)*(h(i))^2/6*(X(i+1)-(X(i)+X(i+1))/2)/h(i)+Y(i+1) -M(i+1)*(h(i))^2/6*(X(i)+X(i+1))/2-X(i))/h(i);
end
disp(Q)
for i=1:n-1
x=X(i):0.001:X(i+1);
z=H(i)*x.^3+l(i)*x.^2+J(i)*x+K(i);
plot(x,z,X,Y,'*');
hold on
end
```





```
>> X=[0 1 2 3 4 5 6 7 8 9 10];  
>> Y=[0.0 0.79 1.53 2.19 2.71 3.03 3.27 2.89 3.06 3.19 3.29];  
>> sanciyangtiao8(X,Y,0.8,0.2)  
    0.3986    1.1684    1.8715    2.4782    2.8733    3.2137    3.0844    2.9199    3.1498  
    3.2223
```





```
function Sanciyangtiao( X, Y )
n = length( X );
for i = 1 : n-1
    h(i) = X(i+1)-X(i);
end
for j = 2 : n-1
    u(j) = h(j-1)/( h(j-1) + h(j) );
    t(j) = 1 - u(j);
    d(j) = 6*( (Y(j+1) - Y(j))/h(j) - (Y(j) - Y(j-1))/h(j-1) )/( h(j-1) + h(j) );
end
t(1)=0;u(n)=0;
d(1) = 0;
d(n) = 0;
for i = 1 : n
    A(i,i)=2; A(1,2)=0; A(n,n-1)=0;
    for j = 2 : n-1
        A(j,j-1)=u(j); A(j,j+1)=t(j);
        if abs(i-j)>1
            A(i,j) = 0;
        end
    end
end
end
```



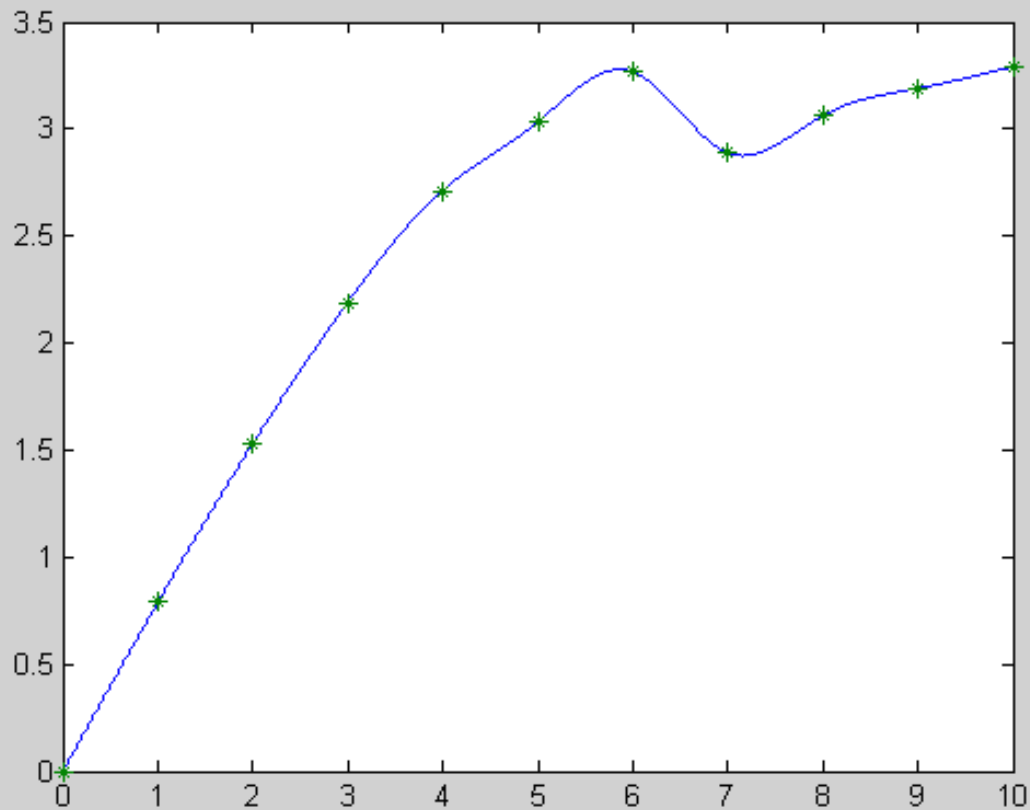


```
D = d';
N = A\D;
M = N';
for i = 1 : n-1
    H(i)=(M(i+1)-M(i))/(6*h(i));
    l(i)=(M(i)*X(i+1)-M(i+1)*X(i))/(2*h(i));
    J(i)=(M(i+1)*X(i)^2-M(i)*X(i+1)^2+2*(Y(i+1)-Y(i))+h(i)^2/3*(M(i)-M(i+1)))/(2*h(i));
    K(i)=(M(i)*X(i+1)^3-M(i+1)*X(i)^3+(6*Y(i)-M(i)*h(i)^2)*X(i+1)-(6*Y(i+1)-
M(i+1)*h(i)^2)*X(i))/(6*h(i));
end
for i=1 : n-1
    Q(i)=M(i)*(X(i+1)-(X(i)+X(i+1))/2)^3/(6*h(i))+M(i+1)*((X(i)+X(i+1))/2-
X(i))^3/(6*h(i))+Y(i) -M(i)*(h(i))^2/6*(X(i+1)-(X(i)+X(i+1))/2)/h(i)+(Y(i+1) -
M(i+1)*(h(i))^2/6)*((X(i)+X(i+1))/2-X(i))/h(i);
end
disp(Q)
for i=1:n-1
    x=X(i):0.001:X(i+1);
    z=H(i)*x.^3+l(i)*x.^2+J(i)*x+K(i);
    plot(x,z,X,Y,'*');
    hold on
end
```





```
>> X=[0 1 2 3 4 5 6 7 8 9 10];  
>> Y=[0.0 0.79 1.53 2.19 2.71 3.03 3.27 2.89 3.06 3.19 3.29];  
>> sanciyangtiao2(X,Y)  
    0.3984    1.1685    1.8715    2.4782    2.8733    3.2138    3.0841    2.9209    3.1459  
3.2368
```







下面5张是第三题





对数据作三次多项式的拟合得到多项式为：

$$f(x) = 0.5491 - 3.9683 \times 10^{-5} x - 2.9977 x^2 + 1.9991 x^3$$

拟合多项式的平方误差为：

$$\text{err}=0.2500$$





多项式拟合的程序如下：

```
function z=f(x,y)
% 二次表达式，按行或按列向量输入x和y.
n=length(x);
sx=sum(x);sx2=sum(x.^2);sx3=sum(x.^3);
sx4=sum(x.^4);sx5=sum(x.^5);sx6=sum(x.^6);
sy=sum(y);sxy=sum(x.*y);sx2y=sum(x.*x.*y);sx3y=sum(x.*x.*x.*y);

A=[ n  sx  sx2  sx3,
    sx  sx2  sx3  sx4,
    sx2  sx3  sx4  sx5,
    sx3  sx4  sx5  sx6];

r=[sy  sxy  sx2y  sx3y]';

t=A\r;
a=t(1), b=t(2), c=t(3), d=t(4)
table=[x y (a+b*x+c*x.^2+d*x.^3) (y-(a+b*x+c*x.^2+d*x.^3))]
disp('x y (a+b*x+c*x.^2+d*x.^3) (y-(a+b*x+c*x.^2+d*x.^3))')
err=sum(table(4).^2)
```





输入:

$x = [-1.0, -0.5, 0.0, 0.5, 1.0, 1.5, 2.0];$

$y = [-4.447, -0.452, 0.551, 0.048, -0.447, 0.549, 4.552]; f(x, y)$

得多项式系数和平方误差为:

$a =$

$0.5491$

$b =$

$-3.9683e-005$

$c =$

$-2.9977$

$d =$

$1.9991$

$err =$

$0.2500$

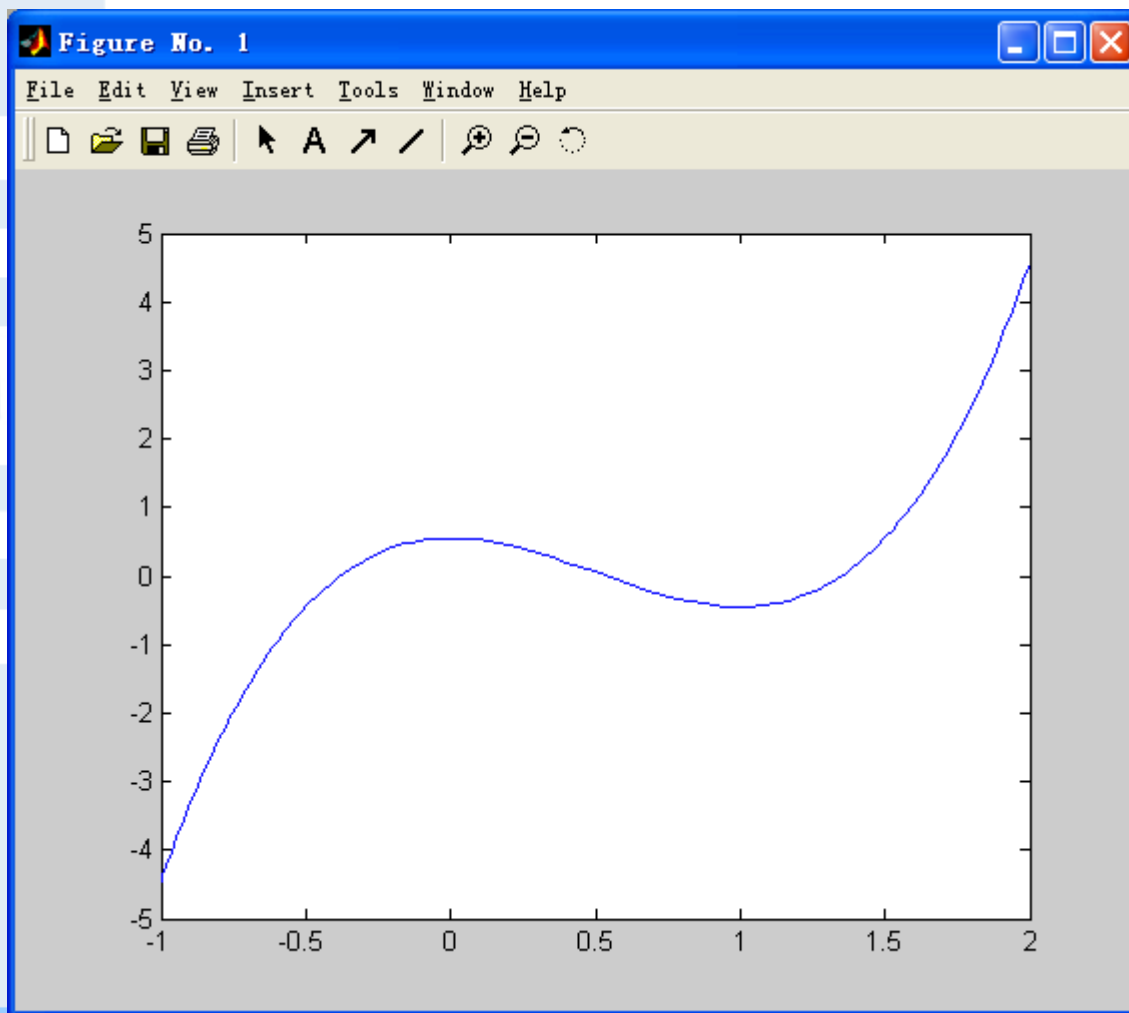




输入下面命令得拟合图形：

```
>> fplot('0.5491+(-3.9683e-005)*x+(-2.9977)*x.^2+1.9991*x.^3',[-1.0,2.0])
```

```
>>
```





输入如下命令得离散数据和拟合曲线的图形：

```
>> x=[-1.0,-0.5,0.0,0.5,1.0,1.5,2.0];y=[-4.447,-0.452,0.551,0.048,-0.447,0.549,4.552];  
>> X=(-1.0:0.001:2.0);Y=[0.5491+(-3.9683e-005)*X+(-2.9977)*X.^2+1.9991*X.^3];  
>> plot(X,Y,'-',x,y,'*')
```

